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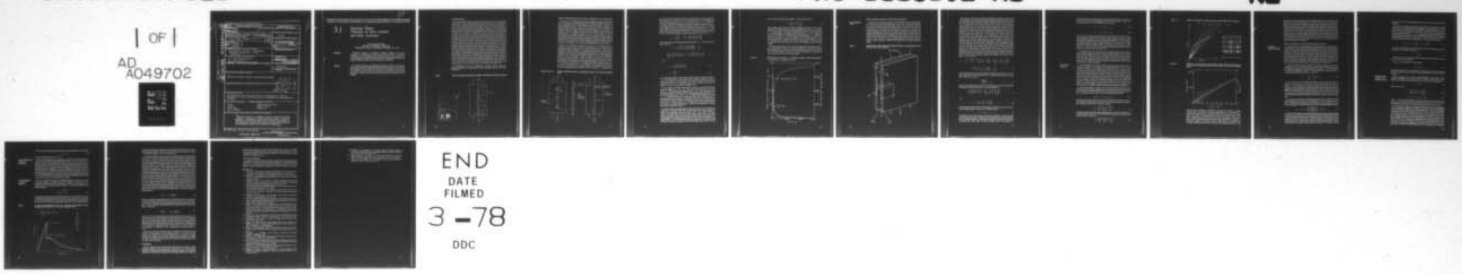

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3.1 Opening Paper: Theories of fibre cement and fibre concrete

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Summary

Theoretical models of cementitious materials reinforced with either continuous or discontinuous, strong and stiff elastic fibres are discussed. Conditions for improved first-crack strength and post-cracking behaviour in such composites are considered. Some uncommon problems encountered in the fracture toughness testing of such composites are elucidated.

Résumé

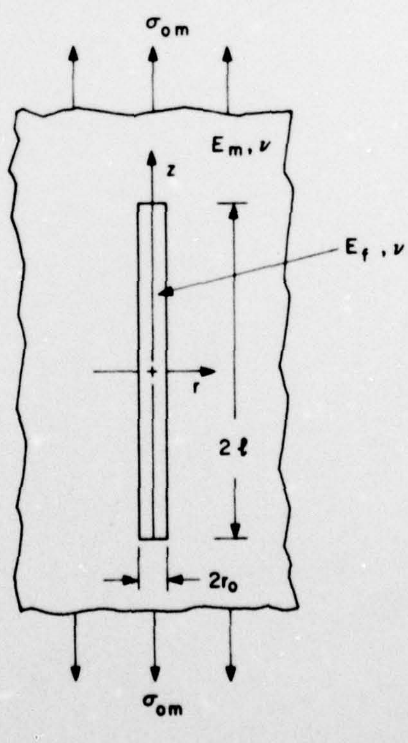
Des modèles théoriques de composites de ciment et de béton contenant des fibres à limite élastique et module d'élasticité élevés sont présentés. Les conditions correspondant à l'amélioration de la résistance à l'initiation des fissures ainsi que le comportement en présence de fissures ont été étudiées. Des problèmes particuliers rencontrés dans les essais de tenacité de tels composites ont été clarifiés.

INTRODUCTION

The reinforcement of relatively brittle construction materials with energy absorbing components is not new. Many examples, such as the use of straw to reinforce mud-bricks, and the present wide-spread practice of the use of steel lath with plaster in interior walls, can be given. Even the use of fibres in concrete is not a novel practice. In fact, the current pre-occupation with reinforcement of cement and concrete with metal and glass fibres is for the purpose of developing an alternative to asbestos fibres which have been used in the past on a large scale (1). As for all applications of composite materials the use of fibres in concrete is largely governed by considerations of economics. Krenchel (1) has recently reviewed the potential of the most promising fibres for application in concrete from both the points of view of technical performance and economics. Taking due note of this constraint, it is nevertheless of interest to assess the theoretical merits of the concepts of fibre reinforcement of brittle materials with high strength elastic fibres. The subject has seen considerable development over the past decade. Romualdi and Batson (2) have shown by an asymptotic fracture mechanics argument that closely spaced fine and very stiff fibres, when incorporated into concrete, can increase the stress at which cracks first appear in a brittle matrix. The same effect has received a different explanation from Aveston, Cooper and Kelly (3) as being due to an insufficiency of releasable elastic energy when the fibre diameter becomes very small while the fibre volume fraction is maintained constant. Most investigators considered effects of fibre aspect ratio, orientation distribution, efficiencies of fibres, effects of interfacial bond strength and many other practical considerations, such as: mixing fibres into wet cement and concrete uniformly, effects of curing time, fibre degradation in wet concrete, etc. These considerations, which are too numerous for us to list here, can be found referred to in several recent reviews and conference proceedings (4-6).

Our considerations here will be limited only to some of the more important theoretical notions in the reinforcement of brittle cementitious matrixes with strong elastic fibres.

Figure 1 **Slender circular fibre in infinite matrix parallel to the principal stress direction in matrix.**

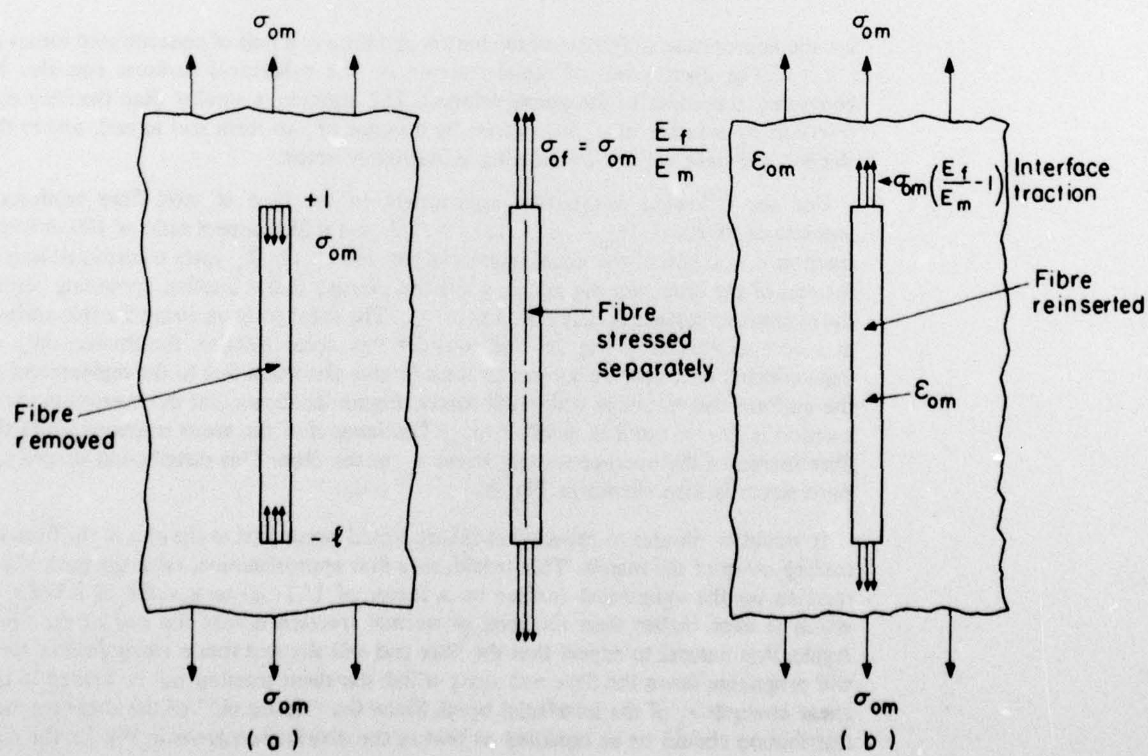


STRESS INHOMOGENEITIES IN FIBRE COMPOSITES AND LAMINATES

The stresses in a composite material are inhomogeneously distributed. Since failure starts from such stress inhomogeneities which are usually accentuated at interfaces, a better understanding of these inhomogeneous stress distributions is essential. In a multi-component material in which each component has different physical and mechanical properties local interfacial stresses arise primarily due to differences in thermal expansion and elastic properties when the temperature of the composite is changed or when it is subjected to external stress. In fibrous composites the stress inhomogeneities of principal importance are those which appear at ends of discontinuous fibres either when the fibre is entirely surrounded by a matrix material or when initially continuous reinforcing fibres fracture under stress. We will be primarily interested here in the interfacial stresses resulting from externally applied stresses for the case of an elastic fibre with a higher modulus embedded in an elastic matrix of somewhat lower modulus—with and without slippage between fibre and matrix.

The geometry of the fundamental problem is shown in Fig 1. A slender fibre of length 2ℓ and radius r_0 having a Young's modulus E_f and Poisson's ratio ν is embedded into an infinite elastic matrix of modulus E_m and the same Poisson's ratio ν. The system is subjected to a uniform tensile stress σ_{om} parallel to the fibre axis. This problem has been considered in various forms with different boundary conditions by many investigators in the past (7-9). An exact solution is difficult to obtain, but a very useful approximate solution is readily obtainable by means of Eshelby's (10) method of analysis of transformation induced stresses. Consider the thought experiment outlined in Fig 2a. One starts by removing the fibre from the matrix and replacing it with perfectly bonded matrix material. A uniaxial tension is then applied producing a uniaxial strain of $\epsilon_{om} = \sigma_{om}/E_m$ and a transverse strain $-\nu\epsilon_{om}$. The matrix material occupying the position of the fibre is now removed but a set of tensile tractions σ_{om} are applied at the two ends of the hole to maintain the strain in the matrix uniform. A stress of $\epsilon_{om} E_f$ is applied to the

Figures 2a and 2b **Eshelby's thought experiment for computing the stresses in and around the cylindrical fibre.**



ends of the fibre which stretches the fibre to make it fit perfectly into the existing hole (Fig 2b). This state of uniform strain in matrix and fibre, however, requires application of internal interface tractions of $\epsilon_{om}(E_f - E_m)$ over an area of πr_o^2 at $z = \pm \ell$. These must be released to obtain the superposition stress field to be added to the applied stress σ_{om}. A very adequate solution is obtained from the theory of elasticity (11) by considering these interface tractions as two equal and opposite point forces applied at $z = \pm \ell$ and acting toward the centre against the joint stiffness of the matrix and the fibre. The full details of this solution can be found elsewhere (12). The important result of interest to us here is that the final interface tractions across the end of the fibre at $z = \pm \ell$.

$$\bar{\sigma}_{zz} = \sigma_{om} \frac{E_f}{E_m} \left[1 - \left(1 - \frac{E_m}{E_f} \right) \left(\frac{K_f}{K_m + K_f} \right) \right] \tag{1}$$

The distributed shear traction along the cylindrical surface $r = r_o$ near the end of the fibre is given by

$$\sigma_{rz} \simeq \sigma_{om} \frac{(r_o/\ell)^3}{8(1-\nu)} \left(\frac{K_m}{K_m + K_f} \right) \left(\frac{E_f}{E_m} - 1 \right) \times$$

$$\left[\frac{(1-2\nu)}{[(r_o/\ell)^2 + (1 - z/\ell)^2]^{3/2}} + \frac{3(1 - z/\ell)^2}{[(r_o/\ell)^2 + (1 - z/\ell)^2]^{5/2}} \right] \tag{2}$$

where

$$K_m = \frac{4\pi r_o E_m (1 - \nu)}{4(1 - \nu^2) - (1 + 2\nu)} \tag{3a}$$

$$K_f = \frac{\pi r_o^2 E_f}{\ell} \tag{3b}$$

are the appropriate stiffnesses of the matrix and fibre to a pair of concentrated forces at $z = \pm \ell$. The distribution of radial traction on the cylindrical surfaces can also be computed according to the above solution. This traction is smaller than the fibre end tractions by a factor of (r_o/ℓ), it varies by a factor of two from end to end, and in the slender rod case we are considering is negligibly small.

For the following parameters appropriate to the case of steel fibre reinforced concrete or mortar, $E_f/E_m \approx 6$ (1, 13), $\nu \approx 1/3$, and a fibre aspect ratio of 100, *a large fraction* $\alpha = 0.966$ of the initial interfacial traction $\sigma_{om}(E_f/E_m)$ gets transmitted across the end of the fibre into the matrix while the average radial traction appearing across the cylindrical surface is only $5.995 \times 10^{-3}\, \sigma_{om}$. The shear traction along the side surface at $r = r_o$ is plotted in Fig 2c. We consider this shear traction distribution only as approximate, as it may be subject to considerable distortion due to the replacement of the uniform end tractions with point forces. Figure 2c shows that the maximum shear traction is at $z = \ell$ and is nearly $0.3\sigma_{om}$. The integral of the shear tractions along the fibre increases the average normal stress σ_{zz} in the fibre. This distribution of average fibre stress is also shown in Fig 2c.

It would be natural to expect that failure would occur first at the end of the fibre by tearing away of the matrix. This would, as a first approximation, raise the peak shear traction on the cylindrical surface by a factor of $1/(1-\alpha)$ to a value of $8.823\sigma_{om}$, which is even higher than the level of normal traction across the end of the fibre. Again, it is natural to expect that the fibre end will slip and that a *shear failure zone* will propagate down the fibre end along which the shear traction will be limited to the shear strength τ_b of the interfacial bond. Since the "elastic tail" of the shear traction distribution should be as confined as that of the distribution given in Fig 2c, the well

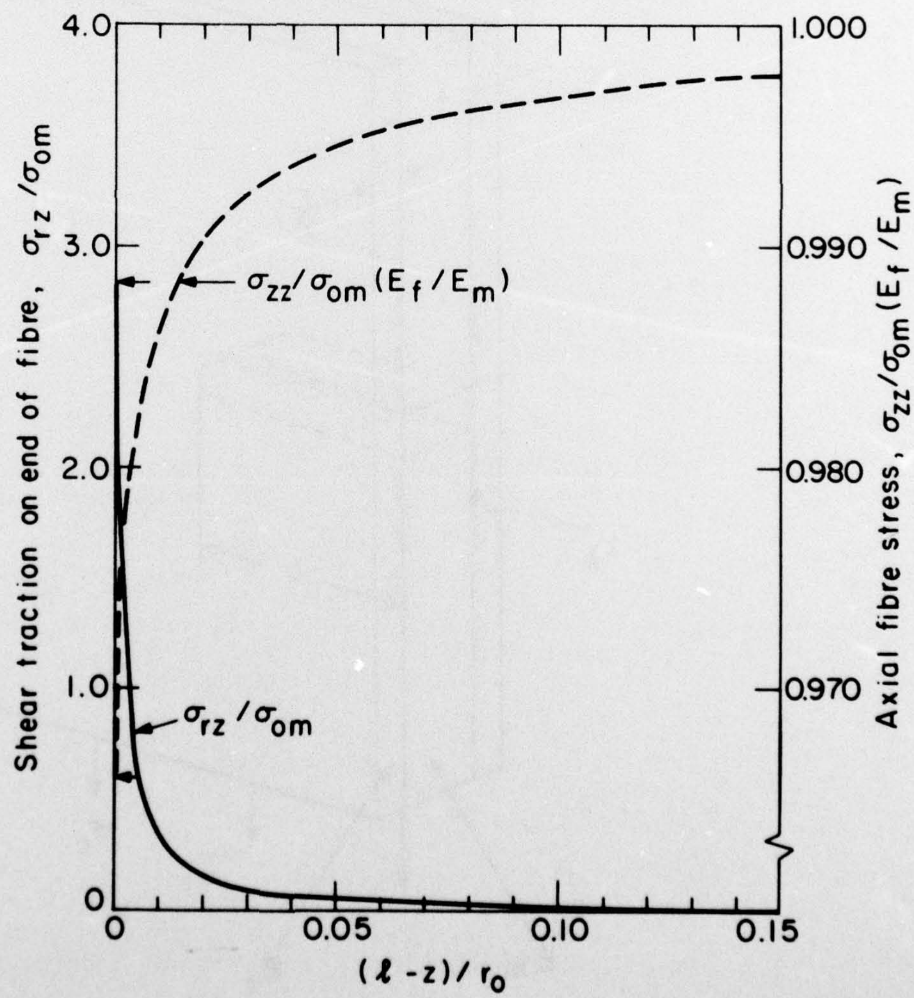
known shear lag estimate for the length λ of the shear failure zone

$$\lambda = \left(\frac{r_o}{2}\ \frac{\sigma_{om}}{\tau_b}\ \frac{E_f}{E_o}\right) \quad . \tag{4}$$

gives a rather accurate value. The relative magnitudes given above remain largely unaffected by the fibre aspect ratio as long as the latter is above 50. If σ_{om} is taken as the first crack strength, in the better examples of fibre reinforced concrete the ratio of σ_{om}/τ_b is around unity (1, 13). This gives a shear failure zone $\lambda \approx 1.5d$ only. Quite often, however, the interfacial bond strength is much lower, giving ratios σ_{om}/τ_b as high as 10, which in turn, result in $\lambda \approx 15d$. The length of the shear failure zone at the first-cracking of the matrix is an important parameter. Thus, if the fibre length is so short or the interfacial bond strength so low that the entire length of the fibre becomes unbonded from both ends, the concrete is likely to fail altogether at the first-cracking strength.

The stresses at fibre ends which occur upon cracking of a reinforcing fibre will be, to a first approximation, similar to the stresses discussed above. These have, however, been investigated in more detail in anisotropic, elastic-plastic situations by McClintock (14). Since such fractures in fibres are of importance primarily in composites with ductile matrixes they will not be discussed here.

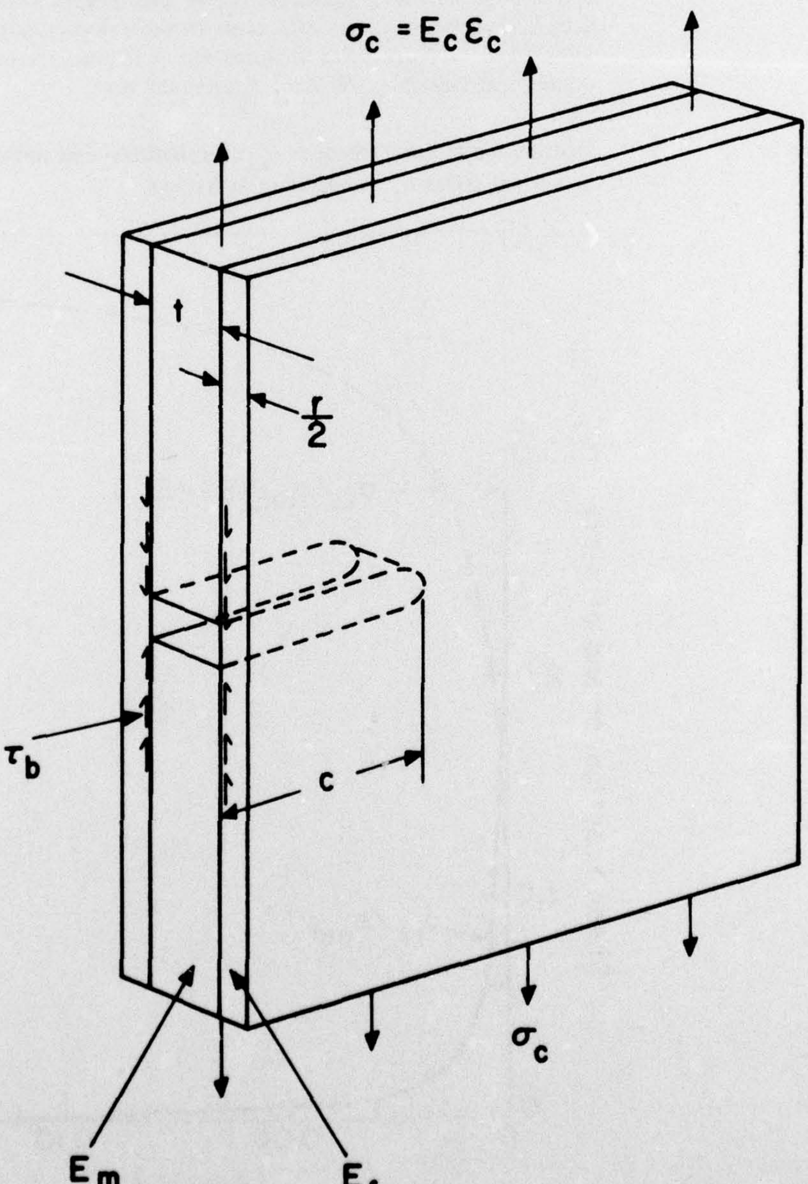
Figure 2c **Distribution of shear tractions σ_{rz} on cylindrical end surface of fibre, and distribution of normal stress σ_{zz} along fibre at its end.**



Non-propagating cracks

Although continuous reinforcing fibres do not lend themselves well for reinforcement of concrete in most conventional applications, some of the more important behaviour can be readily demonstrated in this configuration. One of the most important effects due to incorporation of high strength, high stiffness fibres in concrete (or any other low strength matrix) is the increase they can provide in the first-crack strength of the composite. Romualdi and Batson (2) first pointed out this effect by an argument that strong and stiff fibres tend to decrease the stress intensity factor of matrix cracks and therefore require higher stress to propagate them. Their analysis, which assumed no slippage of fibres, drew attention only to the effect of fibre spacing but failed to indicate the effect of other parameters. A more thorough approximate analysis was given by Aveston, Cooper, and Kelly (3) which we will consider here in a somewhat simplified setting.

Figure 3 **Equivalent three element laminate representing the details for non-propagating cracks in continuous fibre elastic composites.**



When matrix cracks first form and have to propagate around fibres, which we take for this idealization to be continuous and aligned parallel to the direction of stress, they will locally release some stored elastic energy in the fractured matrix. Such cracks will also: a) propagate shear failure zones up along the fibres and thereby do both dissipative "plastic" sliding work and increase the local elastic energy stored in the surrounding reinforcing elements, b) form new surfaces in the cracked matrix, and finally c) due to the increased compliance of the cracked composite, permit the applied stresses to do additional work on the composite. For the quantitative description of this process consider the equivalent laminate shown in Fig 3, where we have lumped together all of the weak, elastic matrix of modulus E_m and volume fraction V_m into an interior layer and lumped all of the reinforcement as two symmetrically placed surface layers of modulus E_f and combined volume fraction V_f. In this model the cracks in the matrix can only run in the matrix along the width of the laminate. The figure shows that in the region where the crack has run a shear failure zone has propagated vertically up and down between the matrix and the reinforcement. All the components of energy and work consisting of the released energy in the fractured matrix, the additional external work on the laminate, the local dissipative plastic shear work at the sliding interlayer, the additional elastic energy stored in the overstressed portions of the reinforcement, and the energy of the fractured surfaces can be readily evaluated by elementary analysis (for the details of the equivalent treatment for fibres see Aveston et al (3)) and the total free energy change for the laminate with a crack of total length c can be written as:

$$\Delta G = c\left[-\frac{E_m^2 t^2 \epsilon_c^3}{3\tau_b} - \frac{E_m^2 t^2 \epsilon_c^3}{2\tau_b}\left(1+\frac{t}{r}\frac{E_m}{E_f}\right) + \frac{E_m^2 t^2 \epsilon_c^3}{6\tau_b}\left(1+\frac{t}{r}\frac{E_m}{E_f}\right)\right.$$

$$\left. + \frac{E_m^2\, t^2\, \epsilon_c^3}{2\tau_b}\left(1+\frac{1}{3}\frac{t}{r}\frac{E_m}{E_f}\right) + 2t\alpha\right] \qquad (5)$$

where the individual terms are those identified immediately above, and where ϵ_c is the distant uniform strain in the laminate and α the specific fracture work in the matrix alone. The crack runs when:

$$\left(\frac{\partial \Delta G}{\partial c}\right)_{\epsilon_c} = 0 \qquad (6)$$

which is only possible when all the terms in the brackets sum to zero. This determines immediately the required elastic strain ϵ_c for crack propagation as

$$\epsilon_c = \left(\frac{12\alpha\,\tau_b\,E_f\,V_f}{E_m^{2}\,E_c\,t}\right)^{1/3} = \left(\frac{12\alpha\,\tau_b E_f V_f^2}{E_m^2\;E_c\;r(1-V_f)}\right)^{1/3} \qquad (7)$$

where

$$E_c = E_f V_f \left(1+\frac{t}{r}\frac{E_m}{E_f}\right) \qquad (8)$$

is the effective Young's modulus of the laminate. Clearly then cracks in the matrix cannot propagate before the stress on the laminate reaches

$$\sigma_c = E_c\,\epsilon_c\,. \qquad (9)$$

The strength of the composite given by eqns. (9) and (7) goes to zero when the fibre volume fraction goes to zero. In reality the tensile strength of the composite will level off at the inherent strength $f_m^{\prime}$ of the concrete itself which is thought to be governed by pre-existing flaws which might be either pre-existing pores, weak interfaces or

shrinkage cracks. Since the introduction of very small volume fractions of fibre is not likely to alter the nature of these inherent flaws the first crack strength f_{fc} would not be merely the sum of f_m and σ_c but the bigger of the two. Hence,

$$f_{fc} = f_m \quad (\text{for } f_m > \sigma_c) \tag{10a}$$

$$f_{fc} = E_c \epsilon_c \quad (\text{for } \sigma_c > f_m). \tag{10b}$$

We see from eqns. (10b) and (7) that for given properties of matrix and reinforcement and composition, the first-crack strength of a fibre reinforced concrete increases proportional to the negative 1/3 power of the thickness dimensions of the reinforcement in qualitative support of the discovery of Romualdi and Batson (2), but increases also with increasing interface shear strength and modulus of reinforcement. A hidden assumption in the above analysis is that the reinforcement is strong enough to support the external stress even after the matrix has fully separated. We will discuss this condition more fully in the next section below.

The statement given in eqn. (5) differs in an important respect from the famous crack instability condition of Griffith (15). In fact, since all energy and work terms are linear in the crack length there is no crack length dependent instability at all. At a stress below that given in eqn. (9) cracks of any length in a matrix, uniformly bridged by unfractured reinforcing elements will not propagate, and if the matrix is initially free of cracks there is no reason for cracks to appear of any length greater than the mean spacing between reinforcing elements.

Post-cracking behaviour

In the developments below both for continuous and discontinuous reinforcement we will ignore variability in the strength of the reinforcing elements. This is not an important restriction for work hardened metal fibres which can be expected to neck and rupture in a rather narrow stress range. The restriction is also not important for unprotected brittle glass fibres which can be expected to severely degrade in strength down to a uniformly low level as a result of attack by the alkali constituents of the cement matrix. On the other hand, the cement or concrete matrix in a fibre reinforced concrete composite is a composite itself of cement, sand, and other coarser aggregates and must therefore exhibit some variability. Although we will not consider such variability in a rigorous manner, we will note that different portions of the matrix can crack at somewhat different stresses. Hence we will take the strength of the reinforcement f_f as a constant but admit some range for the strength f_m of the matrix.

As has been discussed by Aveston et al (3), certain requirements have to be met if the composite with continuous, aligned fibres has to have any post-cracking behaviour. When the stressed composite reaches its first crack strength the uniform strain in it just prior to cracking is given by eqn. (7), and the following equation holds.

$$V_f \epsilon_c E_f + (1 - V_f) \epsilon_c E_m = f_{fc} \tag{11}$$

where the first term gives the portion of the stress supported by the fibres just prior to the formation of the first crack. If the fibres are not to fracture before even the first-crack strength is reached they must have a strength

$$f_f > E_f \left(\frac{12\alpha \tau_b E_f V_f}{E_m^2 E_c t} \right)^{1/3}. \tag{12}$$

Furthermore, if there is to be any post-cracking behaviour at all the fibre strength must be high enough to support the entire post-cracking load without the benefit of any assistance from the matrix, i.e.

$$f_f > \frac{E_c}{V_f} \left(\frac{12\alpha \tau_b E_f V_f}{E_m^2 E_c t} \right)^{1/3}. \tag{13}$$

Figure 4a Model of a progressively cracking continuous element, elastic, fibrous composite.

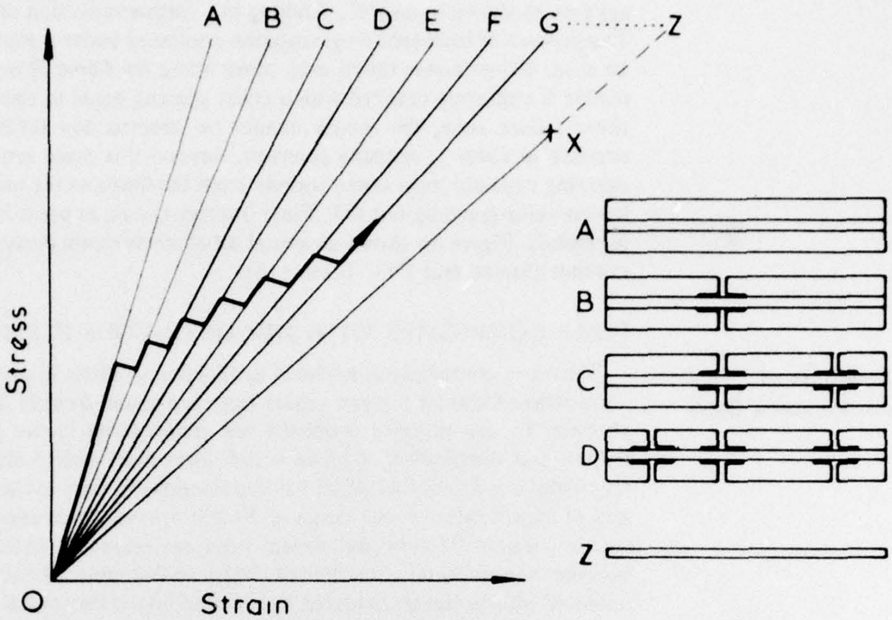
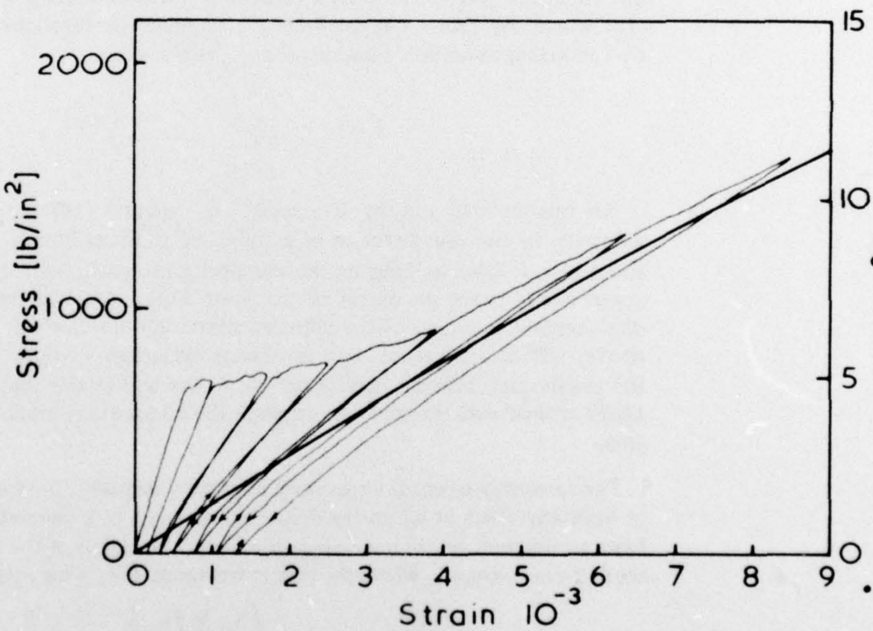


Figure 4b Behaviour of a PVC-gypsum plaster composite under cyclic tensile extensions of progressively increasing stress (from Allen 1971, courtesy IPC Science and Technology Press).



Once the above conditions are satisfied, the composite will continue to support additional load in the presence of increasing fragmentation of the brittle matrix. The process is characterized in Fig 4a, showing on the right inset four stages of the cracking matrix (16). In state A the composite is still intact and its effective modulus is high, giving a loading behaviour shown as line OA. The first crack appears in the matrix as shown in inset B, producing the dark shear failure zone and resulting in a reduction of the effective modulus to a line given by OB. A slight increase in the applied force produces a slight extension in the first shear failure zones along the fibres next to the

first crack, this increases the stress in the remaining matrix of the composite in the regions away from the unloaded portions around the first crack until a second crack appears as shown in inset C, resulting in a further reduction of modulus to the line OC. The process of increased fragmentation continues under a slightly increasing load until all shear failure zones touch each other along the fibres. Beyond this point, where the matrix is uniformly cracked with a crack spacing equal to about twice the extent of the shear failure zone, the matrix cannot be stressed any further as long as the bond strength in shear τ_b remains constant. Beyond this point any further increase in load carrying capacity must come entirely from the fibres as the modulus has dropped to its lowest value given by line OZ. Final fracture occurs at point X where the fibre strength is reached. Figure 4b shows an actual set of stress strain curves in a composite made of gypsum plaster and PVC fibres (16).

FIBRE COMPOSITES WITH DISCONTINUOUS ELEMENTS

Properties of discontinuous fibres

The more commonly considered application of fibres in concrete is in discontinuous form where fibres of a given aspect ratio are mixed directly into the concrete prior to pouring. In this practice problems are encountered in the proper wetting, uniform mixing and distribution of fibres which have been widely discussed in the literature. Generally it is found that small volume fractions of fibre in the range of several percent and of aspect ratios in the range of 50-100 are readily incorporated into the concrete. As the volume fraction and aspect ratio are increased fibres tend to bundle-up and become non-uniformly distributed. Volume fractions of the order of 0.05-0.1 can, however, still be accommodated in concrete panels by special spray-laying techniques where, of course, only planar isotropy and reinforcement is achieved (17).

In relatively small volume fractions of fibre where proper mixing can be achieved, the fibres tend to have a random orientation in space. Their site of intersection with any randomly passed plane tend to be distributed according to a Poisson distribution (18) where the probability of finding r or fewer intersections in an area having an overall average number of intersections n_o, is given by

$$P(r) = \sum_{\rho = o}^{\rho = r} \frac{n_o^{\rho} \exp(-n_o)}{\rho!} . \tag{14}$$

As was pointed out by Romualdi and Mandel (19) the tensile force carrying efficiency in any one direction of a collection of fibres having random orientations in space is $\eta = 0.41$ as long as the composite remains intact. Furthermore, while the composite is intact the extent of the shear failure zones at the ends of fibres due to displacement incompatibilities between matrix and fibre, which was discussed in detail above, will also decrease with increasing departure of fibre orientation away from the tensile axis. Fibres oriented normal to the tensile axis may have no shear failure zones at their ends as they can transmit the compressive tractions acting across their ends.

For randomly oriented fibres several length dimensions have to be defined. For fibres to have any effect at all on the first-crack strength of a concrete composite they must have a minimum length exceeding four times the length of the shear failure zone λ at the first-crack stress f_{fc} where the matrix fractures. This is the *minimum effective length*.

$$\ell_m = 4\lambda = 2r_o\left(\frac{f_{fc}}{\tau_b}\right)\left(\frac{E_f}{E_c}\right) = 2r_o \frac{E_f}{\tau_b} \epsilon_c \tag{15}$$

It results from the following consideration. When the matrix cracks and fibres bridge across the crack, the *shorter of the two lengths of the fibre threading through the plane* of the crack becomes of importance. A little reflection shows immediately that the most probable value of the shorter portion of the fibre defined by its intersection with the plane of the crack is $\ell/4$. If this length is entirely covered with the *shear failure zone* this end of the fibre will be pulled out upon cracking of the matrix. The composite will then

fracture at the first-crack strength regardless of the actual tensile strength of the fibres themselves.

If the fibres are longer than this minimum length then upon intersection by a matrix crack they can carry additional load. This happens by the propagation of two new sets of shear failure zones along both sides of the fibre, starting from the plane of the crack as shown in the inset of Fig 4a. If the composite is to have any post-cracking behaviour at all, the portion of the stress $(1 - V_f)\epsilon_c E_m$ previously carried by the matrix will have to be transmitted to the fibres by the tractions along these new shear failure zones, giving a total shear failure zone length λ' which must not exceed one-fourth of the fibre length. This defines a minimum effective fibre length ℓ_{mpost} for post-cracking behaviour

$$\ell_{mpost} = 2r_o \frac{f_{fc}}{\tau_b} \frac{1}{V_f} = 2r_o \frac{\epsilon_c E_c}{\tau_b V_f} . \tag{16}$$

To achieve this performance the fibre strength f_f must be high enough to support the entire load on the cracked composite, ie

$$f_f > \frac{f_{fc}}{V_f} . \tag{17}$$

Equations (16) and (17) together prescribe the necessary fibre strength and aspect ratio for post-cracking behaviour.

Finally, when one quarter of the fibre length

$$\frac{\ell}{4} > \frac{r_o}{2} \frac{f_f}{\tau_b} , \tag{18}$$

fibres will be fractured instead of being pulled out when the composite fails. This length has been called the *critical length* for ultimate performance of a fibre reinforced composite.

Strength of brittle composites with discontinuous fibres

When a composite such as concrete with discontinuous strong elastic or work hardened ductile fibres with random orientation is stressed, the first crack strength which will be obtained will still be given by eqn. (9) with minor modifications.

$$f_{fc} = \epsilon_c E_c \tag{9}$$

where, however, now

$$\epsilon_c = \left(\frac{12\alpha \tau_b E_f \eta^2 V_f^2}{E_m^2 E_c r_o (1 - \eta V_f)} \right)^{1/3} \tag{19}$$

$$E_c = E_f \eta V_f + E_m (1 - \eta V_f) \tag{20}$$

where $\eta = 0.41$ is the fibre efficiency factor discussed above. This strength will be reached, provided the fibre aspect ratio exceeds the value given in eqn. (15), and the fibre strength is high enough so that premature fibre fracture does not occur.

After formation of cracks in the composite the additional extension of the composite due to its increased compliance will straighten out the parts of all fibres bridging the cracks (except perhaps those which were nearly parallel to the crack). This will produce a marked increase in the fibre efficiency factor from 0.41 to a value much nearer to unity. Hence the cracked composite will be able to support additional load without much increased cracking by a more efficient sharing of the load between fibres. Thus, upon cracking, the discontinuous fibre composite tends to improve its performance by acting somewhat like a composite with aligned fibres. If the conditions given by eqns. (16) and (17) are met, the composite will demonstrate post-cracking behaviour with rising stress. For composites having fibres of aspect ratio larger than the critical value given by eqn. (18) the post-cracking behaviour will tend to approach that of composites

with continuous and aligned elements, giving an ultimate strength for the composite

$$f_{cu} = \eta' V_f f_f \tag{21}$$

where, as mentioned above, $0.41 < \eta' < 1.0$.

Traction displacement relations for fibre pull-out

When tension experiments are performed on a typical steel fibre bearing concrete sample in a relatively stiff testing machine load extension curves of the type shown in Fig 5 are obtained. When the first crack strength is reached and the specimen gains in compliance there is a drop of load as part of the testing system unloads. Upon further extension the curve may rise again. If the conditions in eqns. (16) and (17) are met, a new maximum load exceeding the first-crack strength can be obtained. Once the maximum load point is reached in a composite where the fibre length is less than the critical value given in eqn. (18), the shorter ends of the fibres will begin to be pulled out under a decreasing load for a displacement range not exceeding half the fibre length –with the initial slope of the decreasing portion of the force-displacement curve cutting the axis at a point near one-fourth of the fibre length.

Crack propagation and fracture toughness

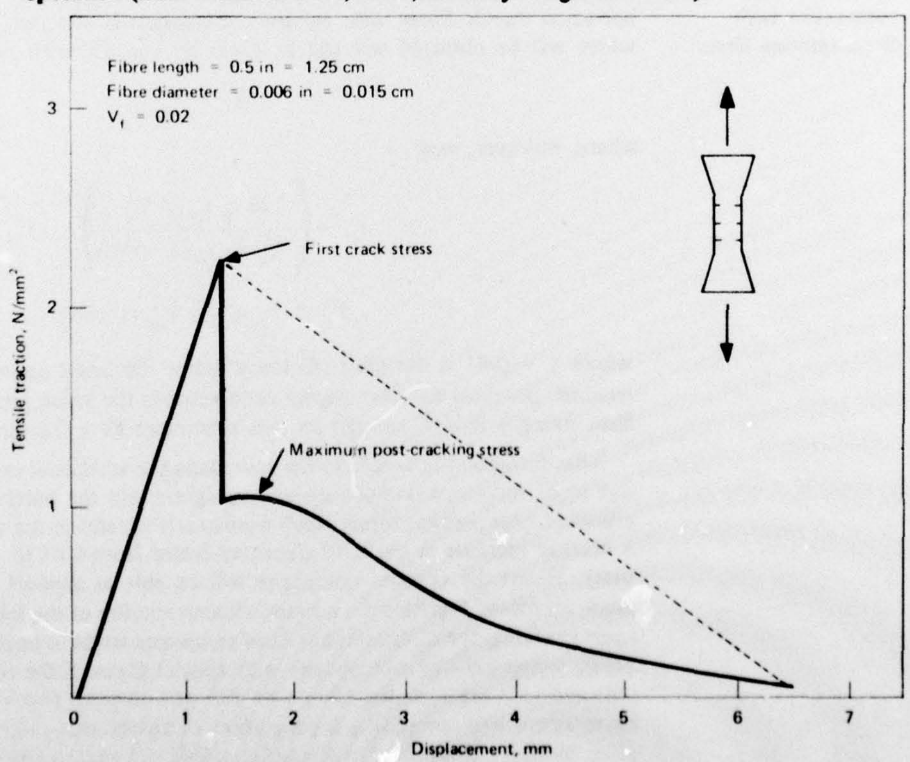
One of the ultimate reasons for incorportating fibres into concrete is to increase its fracture toughness. In materials which have substantially a linear behaviour it is customary to represent the fracture toughness by either the critical mode I stress intensity factor K_{IC} or the specific work for fracture γ_f. The two are related by the well known expression

$$K_{IC} = \sqrt{\frac{2E\,\gamma_f}{1-\nu^2}} \tag{22}$$

for plane strain. Generally the critical stress intensity factor is obtained from standard notched bar tests, while the specific work for fracture can be taken to be the integral of the traction-displacement law discussed in the previous section. Several investigators (see eg Harris et al (20)) have reported that the K_{IC} and γ_f values obtained for fibre

Figure 5

A typical traction displacement curve for a short steel fibre reinforced concrete specimen (from Naaman et al, 1973, courtesy Pergamon Press).



reinforced concrete are not related by eqn. (22). In general, the improvement in K_{IC} due to incorporation of fibres in concrete is much less than the measured improvement in γ_f. The problem appears to be one of specimen size.

Andersson and Bergkvist (21) have recently considered the problem of the instability of a crack in a material having a triangular traction displacement law plotted by the dotted lines in Fig 5 for comparison with the actual law. Their numerical solution shows that as a sample with a crack is loaded, the stress distribution ahead of the crack first starts rising according to linear elastic theory until the crack opening displacement brings the traction to its maximum value. Any further increase in applied stress producing further increases in crack opening displacement will produce a decrease in the traction transmitted across the crack front as it steadily moves the point of maximum traction away from the tip of the crack into the interior. The traction at the tip of the crack finally drops to zero when the crack opening displacement reaches the terminal value shown in Fig 5 where fibres are fully pulled out. At this stage the traction maximum has been pushed ahead of the crack to a distance Δ, and the traction acting across the extension of the plane of the crack between the crack tip and the point a distance Δ away is an inverted and somewhat distorted map of the decreasing portion of the traction displacement law. Andersson and Bergkvist demonstrate that at this point the crack becomes unstable and can propagate under decreasing applied stress. Clearly, the zone of extent Δ ahead of the crack acts in the manner of a "plastic process zone", and the critical crack opening displacement CCOD is somewhere between one quarter and one half of the fibre length. Proceeding further, Andersson and Bergkvist demonstrate that

$$K_{IC} = C_1 \sqrt{(CCOD)} \qquad (23)$$

where C_1 is a constant of proportionality. Since the specific fracture work is a product of one half the maximum of the fibre pull-out traction and the CCOD, it is clear that eqn. (22) also holds. Furthermore, however, an approximate expression between the process zone Δ and the CCOD can be developed from the published results of these authors, which is of the form

$$\frac{\Delta}{CCOD} = \frac{1}{A - (CCOD)/c_o} \qquad (24)$$

where $A = 0.45$ is a constant for one set of numerical studies of the above authors. It is clear, therefore, that for crack lengths of the order of fibre lengths the "plastic process zone" may be many times the CCOD or the fibre length. For the fracture mechanics approach, or the more proper analysis of Andersson and Bergkvist to be applicable, the width dimension of the notched specimens should be many times the combined length of the crack and the "plastic process zone". This condition has not been achieved by any investigator. It is therefore not very surprising that the reported discrepancy occurs.

Naaman et al (18) have considered the size of the largest of the randomly occurring regions of zero fibre density in a fibre reinforced concrete as an initial crack in the concrete, and together with the traction displacement law, prescribe its strength. Although this is an attractive notion which brings in some understanding of a possible size effect, our arguments above show that this concept too would be useful only in very large concrete structures.

DISCUSSION

Above we reviewed some basic concepts which govern the strength of brittle substances reinforced by strong, stiff, and non-brittle fibres, such as work hardened steel. Other, more ductile and extensible fibres, among them polymers, have also been considered as reinforcement for concrete. Our discussion, which does not allow for

plastic extension of fibres would, of course, not apply to these systems. The problems of cost and manufacturing which have been dealt with extensively by other investigators are likely to be of as great importance as the purely technical aspects which we have discussed above.

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